

**SUCCESS**  
**ACADEMY**  
**EDUCATION**  
**INSTITUTE**

**Connecting Representations**  
Adapted from [Fostering Math Practices](#)

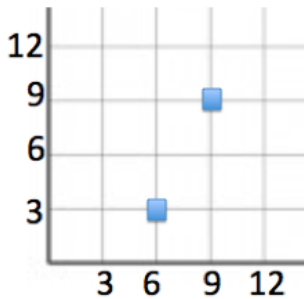
**Grade 5**

The purpose of this Mini-lesson is to reinforce the importance of looking carefully at the scale of the axes when finding distances on the coordinate plane. Show scholars all representations below. Ask them which description and coordinates match which points on the graph. Give scholars time to think independently and discuss with a partner before sharing out. Once all scholars are convinced of the representations that match, have them write the coordinates and a description of the remaining point and plot the point with the remaining coordinates on the graph.

**Key Questions:**

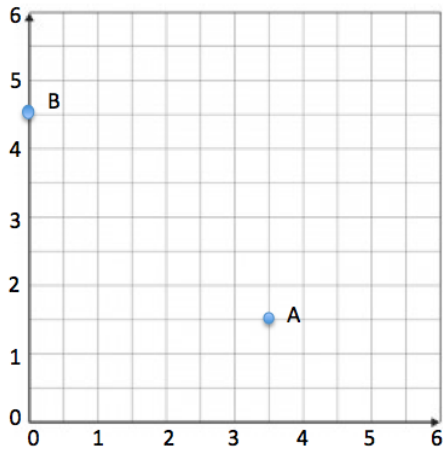
- Which description matches each point? How do you know?
- How are the coordinates of a point related to its distance from the x- and y- axes?
- What is the distance between each grid-line? How do you know?
- Why is it important to look carefully at the axes before finding distances on the coordinate plane?

**Example 1**



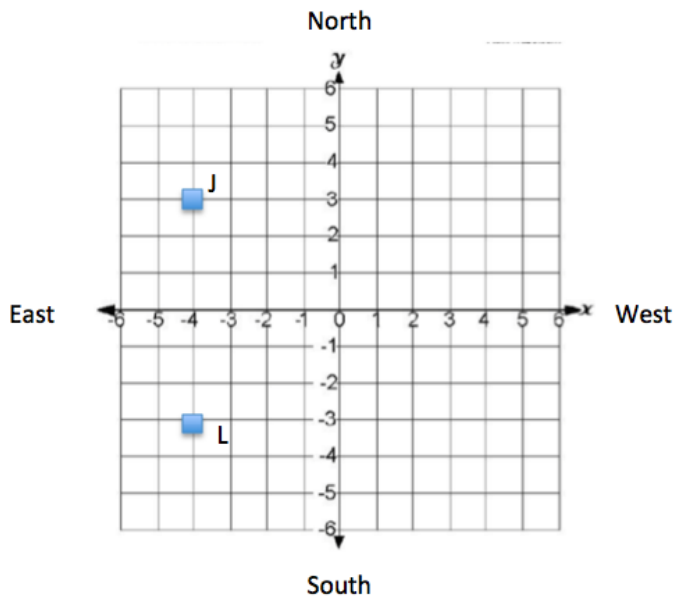
- 6 units from the y- axis and 3 units from the x- axis
- (9, 9)
- (0, 12)

**Example 2**



- A point that is zero units away from the y- axis and 4.5 units away from the x- axis
- $(3.5, 1\frac{1}{2})$

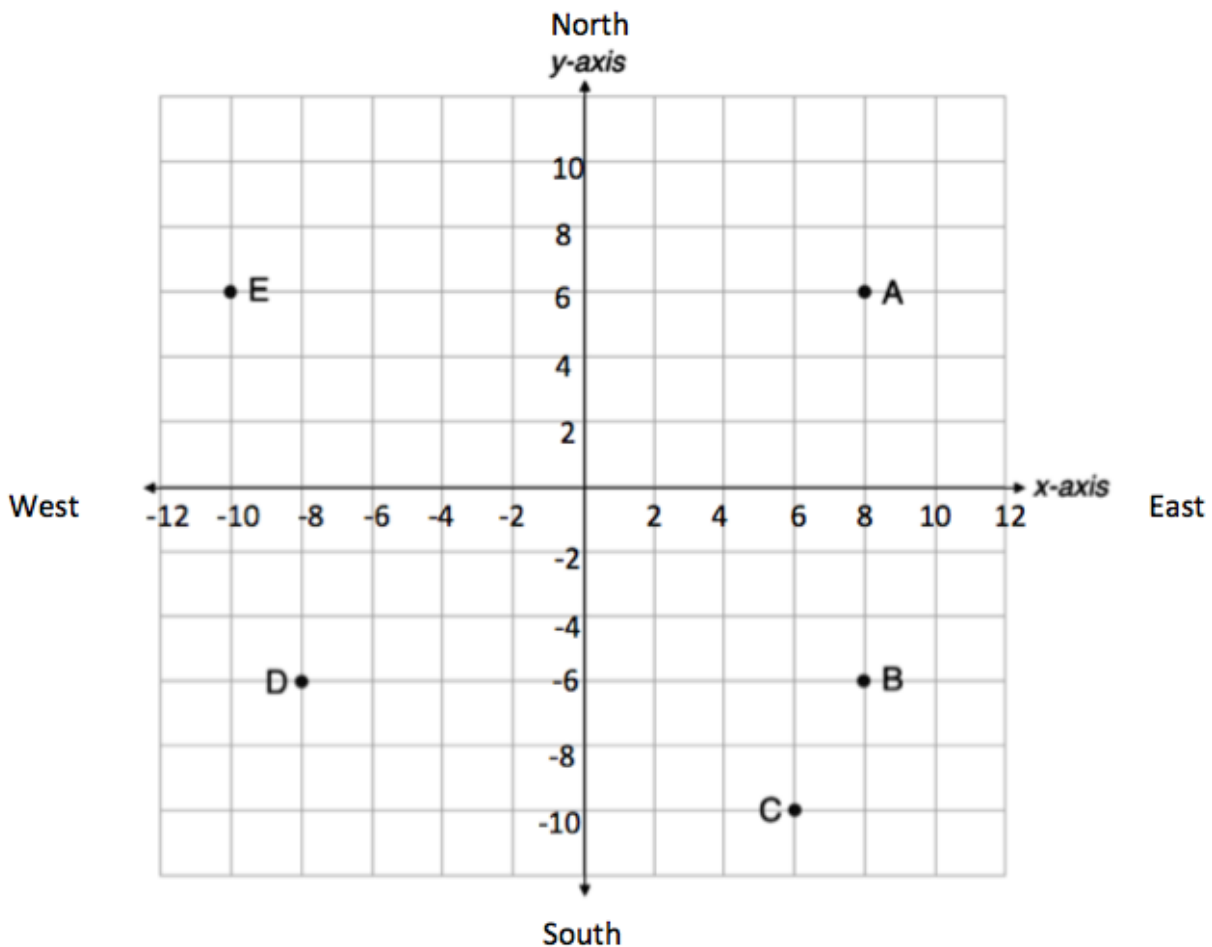
### Example 3



Martine has used a coordinate plane to model a soccer field to show her teammates some plays she has planned for their next game. The points plotted above represent some of the players on Martine's team. Determine which description matches which point, and plot the point that is missing.

- 6 units west of a teammate, and 3 units north of the x- axis
- $(-4, 3)$
- 3 units south of the x- axis and four units east of the y- axis

### Example 4



- $(-8, -6)$
- 6 units north of the x- axis and 8 units west of the y- axis
- $(6, -10)$
- 8 units east of the y- axis and 6 units north of the x axis
- $(8, -6)$
- What is the distance between point B and point A?

## Grade 6

### Example 1

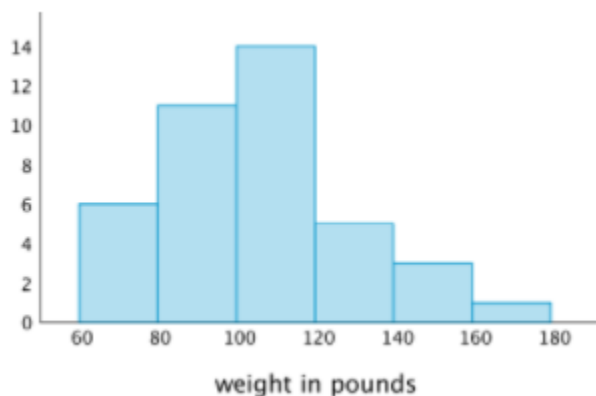
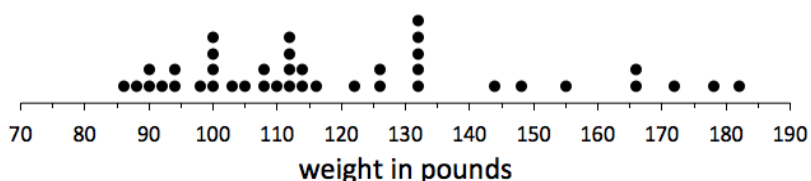
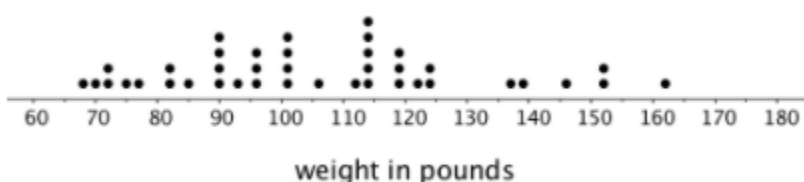
The purpose of this Mini-lesson is to review how to read line plots or dot plots from Grade 5, and to introduce histograms as another way of representing similar information. Use this Mini-lesson to define the term **histogram** and to discuss key features that differentiate histograms from line plots or dot plots. In particular, histograms tell you the frequency of data points within a certain range, but they don't tell you exactly what those data points are.

Scholars learned how to create and interpret line plots in Grade 5, but they may need a reminder. Clarify for scholars that these dot plots are the same as the line plots they saw in Grade 5; they just use different symbols. Use this time to ensure that scholars can accurately read the number of data points, the frequency of each value, and the value of each data point from the line plot before moving on.

Put up the representations and ask scholars which plots could represent the same data sets. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain specific features of the data sets that are or are not the same in the different representations. Once scholars convince one another that the first dot plot matches the histogram, have scholars create their own histogram that matches the second dot plot.

### Representations

These data sets correspond to the weights of dogs at different shelters.



*Note that in this histogram, each range includes the left-end value but not the right-end value. Tell scholars that this is a decision that mathematicians make, and set the expectation that they clarify this when drawing histograms.*

### Key Questions

- Could the two dot plots represent the same data set? How can you be sure?
- This other chart is called a **histogram**. What do you think the numbers along the y-axis of this histogram represent?
- How is this histogram similar to and different from bar graphs that you have seen in the past?
- What do you think are the key features that make a graph a histogram?

- Could this histogram represent the same data set as either of the dot plots? How do you know?
- Is it possible that the histogram represents a different data set from the first dot plot? How could this be?
- How can we create another histogram to represent the data set shown in the second dot plot?

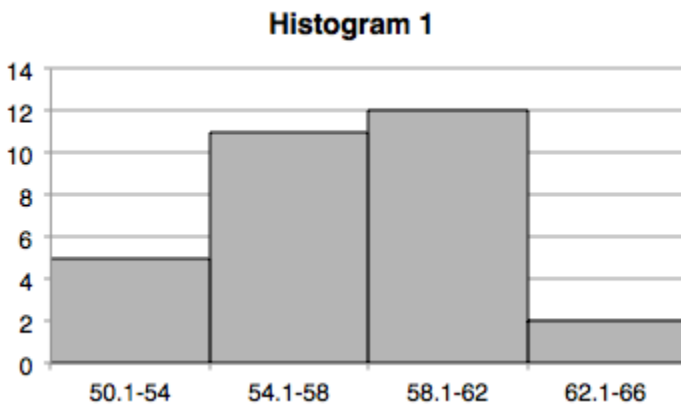
### Example 2

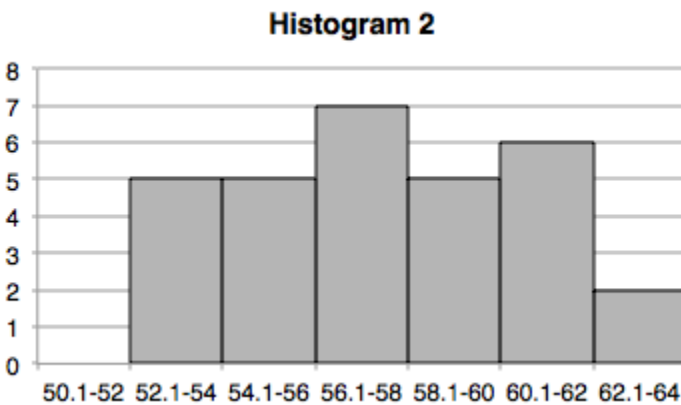
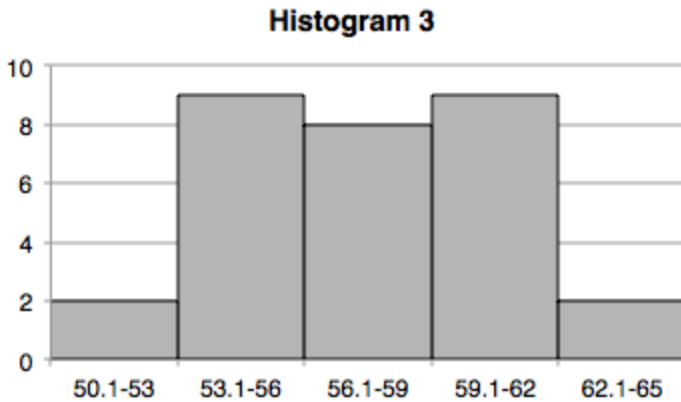
The purpose of this Mini-lesson is to have scholars practice reading and comparing histograms. In addition, this Mini-lesson is designed to get scholars thinking about what information histograms reveal and what is still left uncertain when data sets are displayed in histograms.

Put up the representations and ask scholars which plots could represent the same data set. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain specific features of the data sets that are or are not the same in the different representations. Press scholars to explain whether the data sets must match, could match, or could not possibly match, and to support their reasoning. Once scholars convince each other that Histograms 1 and 3, but not Histogram 2, could represent the same data set (though we don't know for sure) have them invent their own data set that will match both Histogram 1 and 3.

### Representations

These data sets correspond to the average temperatures for each day in a month. Which displays could represent the temperatures for the same month?





#### Key Questions:

- How is the center and spread of each data set similar or different?
- What is the most frequent value (or bin of values) for each data set?
- Which histograms could represent the same data set? How can you be sure?
- Is it possible for those two histograms to represent different data sets? Why or why not?
- Which histograms could **not** represent the same data set? How can you prove that?
- How can we design a data set to show that it is possible that Histogram 1 and 3 represent the same month, but not Histogram 2?
- How can you compare histograms that do not use the same intervals?
- How does changing the intervals or the size of the intervals affect the conclusions you draw about a data set?

#### Example 3

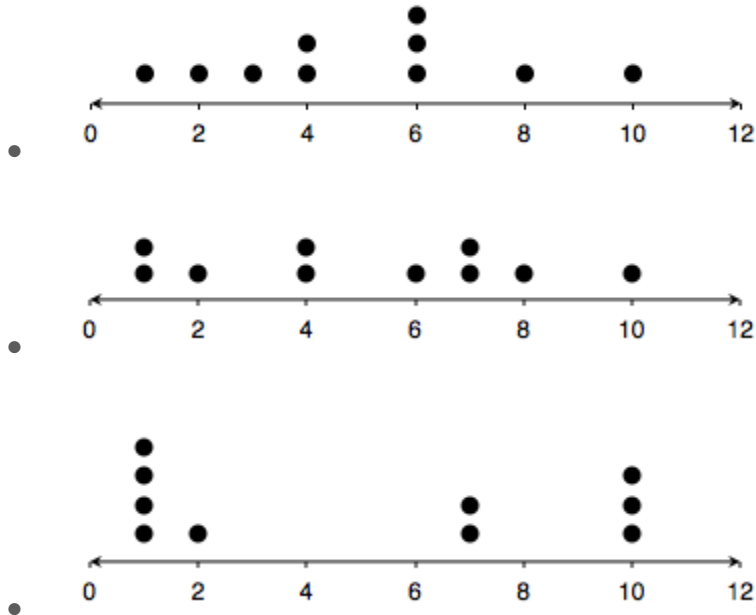
The purpose of this Mini-lesson is to expose scholars to the limits of using mean and range to describe different data sets. This Mini-lesson provides a motivation and justification for exploring the new value you will define with scholars today – the **mean absolute deviation (MAD)**. During this time, your focus should be on solidifying their understanding of mean and range by getting scholars to explain how it's possible for data sets that look so different to have the same mean and range. We do **not** expect scholars to discover the MAD on their own, but we do want to ensure that they understand why the MAD is important and when it is useful.

Put up the representations and ask scholars which descriptions and displays could match. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain specific features of the data sets that are or are not the same in the different representations. Once scholars convince one another that each dot plot matches the mean and

range given, ask them whether they think those measures are good summaries of these data sets, and what features make the data sets different.

### Representations

- A data set has a mean of 5 and a range of 9.



### Key Questions

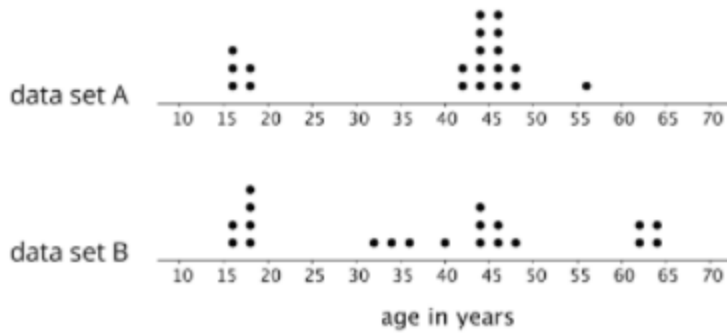
- What is the mean for each data set in the dot plot? How do you know? What does that tell you about the data set?
- For which dot plot do you think the mean is a good representation of a typical value? Explain.
- What is the range for each dot plot? What does that tell you about the data set?
- How is it possible that these data sets have the same range? What information is missing when you know the range of a data set?
- How would you describe the variability or variation in the data for each dot plot?
- These data sets look very different, but when we measure the mean and the range, they appear to be the same. Can you think of a way, besides the range, to measure this variation in order to distinguish between these data sets?

### Example 4

The purpose of this lesson is to give scholars practice with estimating mean and MAD visually and interpreting data sets in context. Put up the dot plots and the descriptions and ask scholars which description matches each dot plot. Give scholars time to think independently and discuss with a partner before sharing out. Press them to explain how they can estimate the MAD without calculating, and then have them calculate it to prove their answer.

Once they have paired each dot plot with a situation, have them describe or sketch a data set that would match the remaining description. If time permits, have scholars create a data set with that exact MAD, but if not, have them describe how that data set would compare to the other two.

### Representations



- Twenty people - high school students, parents, guardians, and teachers - attended a rehearsal for a high school musical. The mean age was 38.5 years and the MAD was 16.5 years.
- Family members of the players usually come to watch the high school soccer team practice. One evening, twenty people watched the practice. The mean age was 38.5 years and the MAD was 12.7 years.
- Twenty people watch a chess tournament. The mean age was 38.5 years, but the MAD was 20 years.

**Key Questions:**

- What does MAD stand for? What does it represent or tell us about a data set?
- How can you compare the MAD of these data sets without calculating?
- How can you prove that this description matches this data set?
- Do data sets with larger ranges always have larger MADs too? Explain your reasoning.
- Which description does not match either data set? How do you know?
- How would that data set compare to the two that we have here? How could you create your own data set to match that description?

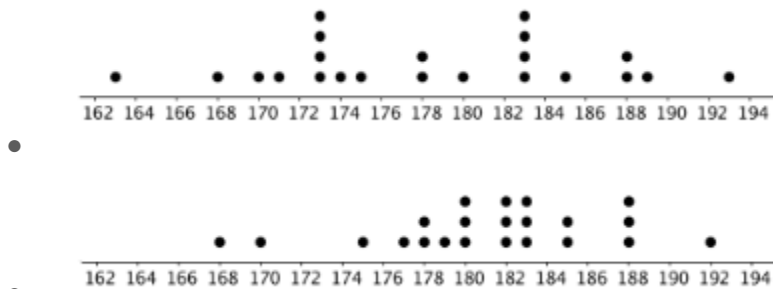
**Example 5**

The goal of this Mini-lesson is for scholars to practice finding the median and using the mean and the median to describe a “typical” value of a data set.

Put up the representations and medians and ask scholars which ones match. Give scholars time to think independently then discuss with a partner before sharing out.

**Representations**

These dot plots represent the heights, in centimeters, of the the first 22 presidents and the next 22 presidents.



- Median = 178 centimeters
- Median = 182 centimeters



### Key Questions:

- Without calculating, which dot plot do you think has the greater median?
- How can you find the median of these dot plots?
- How would the median of each data set change if you remove the minimum value?
- What is the mean for each data set, and how does it compare to the median?
- Which measure do you think is a better representation of a “typical” value for each data set? Why?

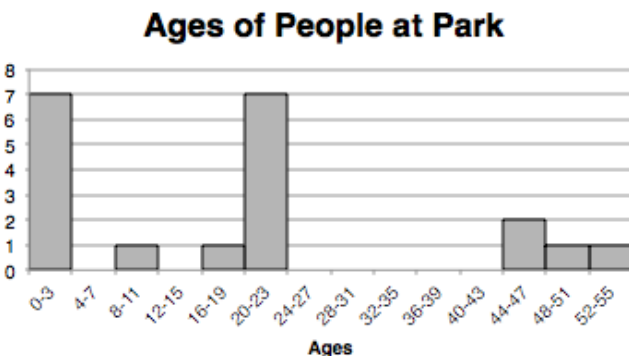
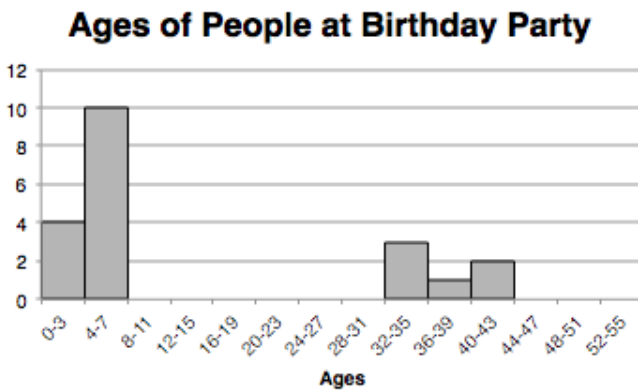
### Example 6

The purpose of this Mini-lesson is to have scholars practice interpreting histograms and finding median and interquartile ranges (IQR). Focus your discussion on what scholars can infer from the histograms, and what they still don't know about these data sets.

Show scholars one of the histograms first and ask whether it is possible to calculate the median exactly from this representation, and if not, how scholars can use this representation to estimate the median. Once scholars have reasoned about how to use the frequencies to estimate the median, show all the representations and ask them which description matches which histogram. Give them time to think independently and discuss with a partner before sharing, and press scholars to explain how they can estimate the IQR for each histogram.

### Representations

Tell scholars that the histograms below show the ages of people at a birthday party and at a park.



- Median age = 20 years and IQR = 22 years
- Median age = 5 years and IQR = 30 years

### Key Questions:

- Can you find the exact IQR from a histogram? Why or why not?

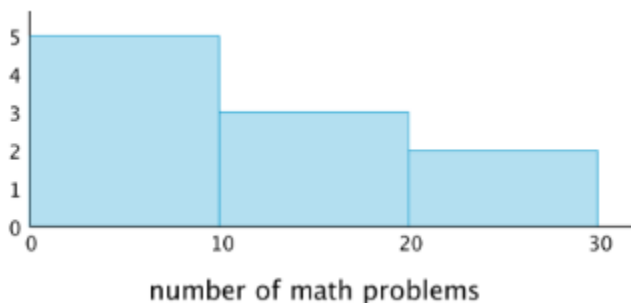
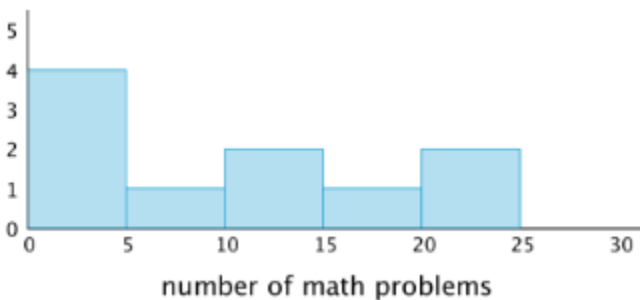
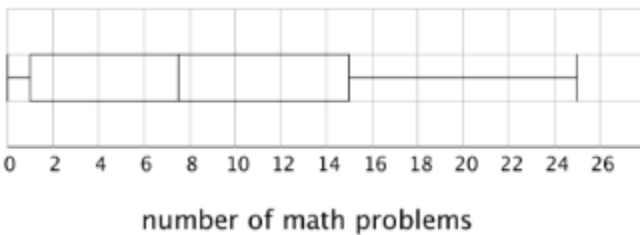
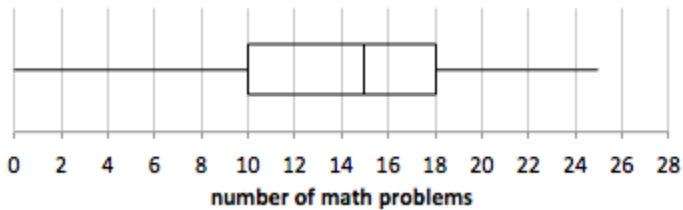
- How can you estimate the minimum possible IQR from a histogram?
- How can you estimate the maximum possible IQR from a histogram?
- Do you think it is easier to estimate the mean or median using histograms? Explain your reasoning.
- Do you think it is easier to estimate the MAD or the IQR using histograms? Explain your reasoning.

### Example 7

The purpose of this Mini-lesson is to have scholars practice comparing data sets that are represented in different ways and making inferences about these data sets. Put up the representations and ask scholars which ones could represent the same data set. They should quickly notice that the two histograms could represent the same data set, so press them to explain whether the box plots could also represent these data sets and to use specific features of the representations to justify their answers.

### Representations

Each plot represents the number of math homework problems scholars completed over ten days.



**Key Questions:**

- Is it impossible for any of these representations to match the same data sets? Which ones?
- Is it guaranteed that any of these representations match the same data sets? Why or why not?
- How would you describe the symmetry and spread of these data sets?
- For these data sets, how many data points are in each section of the box plot? How can that help you compare them to the histograms?
- What can you infer about data sets that are represented in box plots? Histograms?
- Can you estimate the mean of these data sets? Which representation is the easiest to use?

**Grade 7**

**Example 1**

The purpose of this routine is to give scholars practice with pinpointing what must match up between complicated situations and the simulations that could be used to analyze them.

Display all situations and simulations, and ask which simulation could represent each situation. Press scholars to use theoretical and experimental probabilities to clearly explain why the simulations match. For the situation that doesn't match either simulation, have scholars design their own simulations and share out.

<b>Situations</b>	<b>Simulations</b>
<ul style="list-style-type: none"><li>● The chance that any child born is a girl is <math>\frac{1}{2}</math>. What is the probability that a person who has 5 grandchildren has no more than 1 granddaughter?</li><li>● An automobile factory noticed a flaw. They estimate that approximately 10% of cars assembled over the last week have defective brakes. Five cars from that factory are shipped to a dealership near your school. What is the probability that no more than one of these cars have defective brakes?</li><li>● The staff at a local park estimate that there is a 40% chance of seeing an eagle on any given day that you visit the park. If your class plans to visit the park on 5 different days, what is the probability that you will see an eagle on 1 or more days?</li></ul>	<ul style="list-style-type: none"><li>● In a bag, there are 9 blue marbles and 1 red marble. Choose a marble at random, record the color, and replace it. Do this 5 times to record 5 colors, then repeat this process many times. Count the number of times you choose 1 red marble or no red marbles out of 5.</li><li>● Make a spinner with 6 blue sections and 4 green sections, all the same size. Spin the spinner 5 times and record the color each time, and repeat this process. Count the number of times you get 1 or more greens.</li></ul>

**Key Questions:**

- What are the possible outcomes of this situation. Are they equally likely?
- What are the possible outcomes in this simulation? Are they equally likely?
- What is similar and different about these two simulations?
- Which outcome or outcomes in the simulation match each outcome in the realistic situation? How do you know?
- How can we design an experiment to match the other realistic situation?
- What would you need to do to find the theoretical probability of each situation?

### Example 2

The purpose of this routine is for scholars to practice explaining how random events can be used to simulate complicated situations. Build on your conversations from Lesson 3 by getting scholars to think about whether they would have an easier time envisioning the possible outcomes and relative likelihood of each for the situations or the simulations, and about how envisioning all possible outcomes changes when the event is repeated. These ideas will be solidified during the Discourse.

Display all situations and simulations, and ask which simulation could represent each situation. For the situation that doesn't match either simulation, have scholars design their own simulations and share out.

<b>Situations</b>	<b>Simulations</b>
<ul style="list-style-type: none"><li>• In a small lake, 25% of the fish are female. When you catch and release 4 fish, what is the probability that exactly 3 fish are female?</li><li>• Elena plays basketball and she makes 80% of the free throws she takes. What is the probability that Elena will make exactly 3 out of her next 4 free throws?</li><li>• Chelsea and Daniella sing in the school choir. Before each concert, there is an equal chance that the choir teacher will select Chelsea or Daniella to sing the solo. What is the probability that Chelsea will be selected to sing the solo at exactly 3 out of 4 concerts?</li></ul>	<ul style="list-style-type: none"><li>• Fill a bag with two red marbles and two blue marbles. Without looking, choose a marble and replace it four times in a row, and record the color each time. Repeat this process many times, and find the fraction of trials in which exactly 3 blue marbles appear.</li><li>• Make a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner 4 times and record the outcomes. Repeat this process many times, and find the fraction of the trials in which a 4 appears exactly 3 times.</li></ul>

### Key Questions:

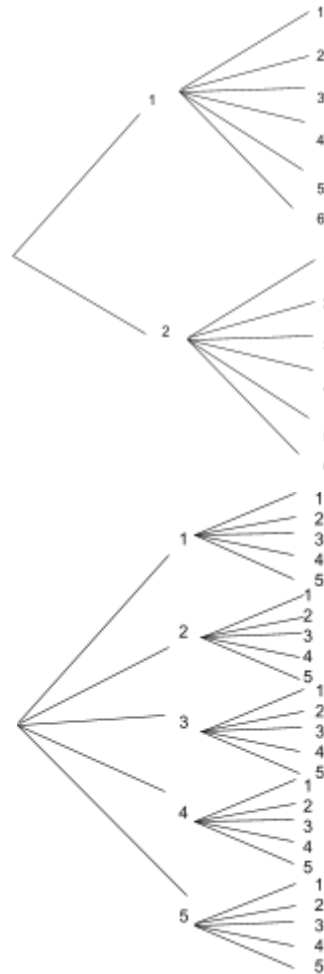
- What are the possible outcomes of this situation? Are they equally likely?
- What are the possible outcomes in this simulation? Are they equally likely?
- Is it easier for you to envision the possible outcomes and the likelihood of each one for the simulations or the situations? Why?
- Which outcome or outcomes in the simulation match each outcome in the realistic situation? How do you know?
- How can we design an experiment to match the other realistic situation?

### Example 3

The purpose of this Mini-lesson is for scholars to practice visualizing sample spaces of compound events and interpreting tree diagrams to determine which ones match. Once scholars agree on which event and sample space match below, have them represent the sample space of the remaining event and describe an event that matches the remaining tree diagram.

<b>Events</b>	<b>Sample Spaces</b>
<ul style="list-style-type: none"><li>• Spin 2 spinners with equal-sized sections labeled 1 through 6.</li></ul>	

- Spin 2 spinners with equal-sized sections labeled 1 through 5.



### Key Questions

- What does each number in this tree diagram represent?
- What outcome does this branch of the tree diagram represent?
- Does the outcome of the first event affect the outcomes of the second event in these cases? How do you know?
- How can you prove whether or not this tree diagram includes all possible outcomes, with none repeated and none missing?

### Example 4


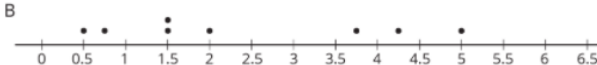
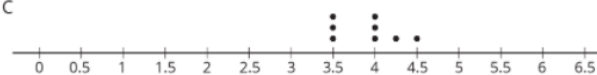
Scholars should have calculators during this time.

The purpose of this Mini-lesson is to review how to calculate mean and mean absolute deviation (MAD). Open by asking scholars what they remember about **mean** and **mean absolute deviation** from Grade 6. At this point, they should say things like, “Mean is the average” and “MAD tells us something about how spread apart the points are.” Put up all five representations and ask scholars to match the numerical values to the corresponding dot plots.

Scholars learned how to calculate these values in Grade 6, but they will likely need a reminder. Push scholars to interpret these values informally (i.e., a larger MAD means more variation) **before** reviewing how to calculate the MAD of the first two data sets (dot plots A and B). Use these

calculations to have scholars evaluate their interpretations, and finish the Mini-lesson by having scholars calculate the mean and MAD of the third dot plot (dot plot C) on their own.

### Representations

<p><b>Dot Plots</b> These data sets correspond to the length, in minutes, of songs on different CDs.</p> <p><b>A</b></p>  <p><b>B</b></p>  <p><b>C</b></p> 	<p><b>Mean and MAD</b></p> <ul style="list-style-type: none"> <li>• Mean: 5.57 minutes and MAD: 0.44 minutes</li> <li>• Mean: 2.41 minutes and MAD: 1.44 minutes</li> </ul>
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### Key Questions

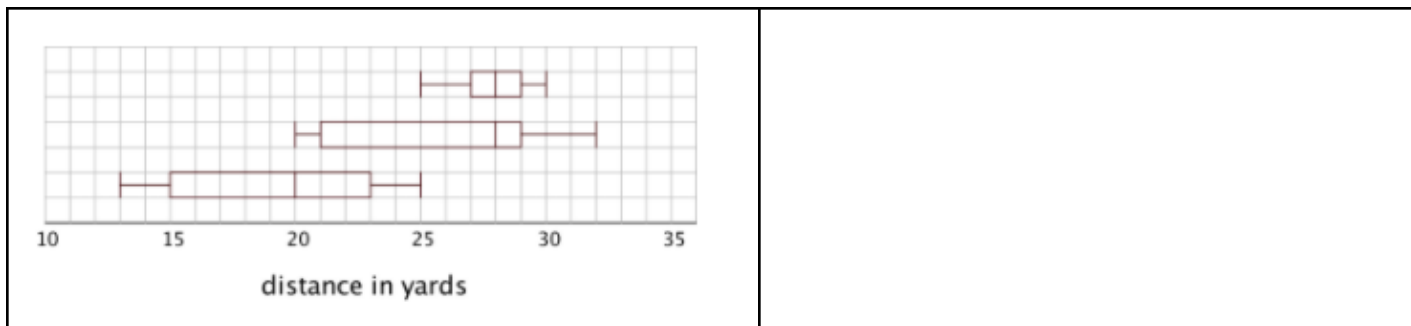
- Describe what one individual dot represents in this context.
- How can you estimate the mean visually without calculating?
- What does the MAD tell us about a data set? How can you tell which data sets have a larger MAD without calculating?
- What conclusions can you draw from the dot plots? What conclusions can you draw when you know the exact mean and MAD?
- How can you calculate the mean and MAD from a data set or dot plot?

### Example 5

The purpose of this Mini-lesson is to review the meaning of **median** and **interquartile range (IQR)**, and how those values are represented in box plots. First, ask scholars what they remember about what the median and interquartile range tell you about a data set. Then show all the representations below and ask them which box plots match which person's data. Finish the Mini-lesson by telling scholars that the third box plot represents Maria's paper airplane and asking them to find the median and IQR from the box plot.

During this discussion, label the median and IQR directly on each box plot and leave this up for your class to reference during the Explore. They will return to analyzing and comparing box plots later on in this lesson.

<p><b>Box Plots</b> Three scholars built paper airplanes and tested to see how far they would fly.</p>	<p><b>Median and IQR</b></p> <ul style="list-style-type: none"> <li>• Andre's airplane: Median distance – 28 meters, IQR – 2 meters</li> <li>• Lin's airplane: Median distance – 28 meters, IQR – 8 meters</li> </ul>
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### Key Questions:

- How is the median similar to and different from the mean?
  - *Make sure to focus this conversation not just around the differences in how these values are calculated, but what those differences mean for how these values are interpreted as well.*
- How is the IQR similar to and different from the MAD? How is it similar to and different from the range?
- Where do you see the median and IQR in each box plot?
- What fraction of trials was above the median? What fraction was below?
- What does the line on the far left represent in each box plot? What does that represent in this context?
- What does the line on the far right represent in each box plot?
- What does the box in the middle represent in each box plot? What fraction of trials fit inside that box?

## Grade 8

### Example 1

The goal of this Mini-lesson is to introduce scholars to **factored form** and to have them articulate how and why the factors of quadratic functions are related to the  $x$ -intercepts. Throughout this lesson, you must stay focused on interpreting factors and  $x$ -intercepts in terms of multiplication. While some scholars might recognize that the zeros of the function are equivalent to  $p$  and  $q$  in the factored form  $y = (x - p)(x - q)$ , it is **much** more important that scholars are able to use an understanding of the meaning of  $x$ -intercepts and the zero property of multiplication to prove their thinking. Because of the extensive precision errors scholars make when subtracting negative numbers, simply memorizing how to use the values of  $p$  and  $q$  to graph a quadratic function is inevitably going to lead to mistakes.

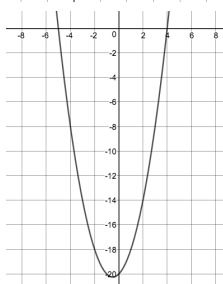
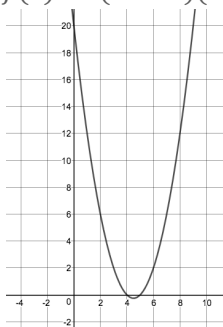
Launch the Mini-lesson by showing scholars the first equation  $y = (x + 4)(x + 5)$  and asking them what type of function they think this will be. Ask them to think about when they have seen similar expressions before, and remind them that they should use the area model to keep track of their work when expanding the expression to rewrite this function.

Once scholars have re-written this function in standard form, ask them what they know about the graph of this function. Give them time to think independently then discuss with a partner before sharing. As they share out, press them to identify specific key features of this graph (the shape,  $y$ -intercept, direction of the parabola, etc) and record their observations. Use this Launch to introduce the term **factored form** to describe the way that this equation is written as a product of two factors. Make sure that scholars understand that a “factor” in this context means the same thing as the “factors” they are used to finding for whole numbers.

Then put up the additional equations and graphs and ask scholars which equations and graphs represent the same functions. Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class. As scholars share out, ensure that they use precise academic language and prove their thinking.

### Representations

- $f(x) = (x + 4)(x + 5)$
- $f(x) = (x + 8)(x + 5)$
- $f(x) = (x - 4)(x + 5)$



### Key Questions

- How can we prove which equation matches which graph?
- How can we use our understanding of  $y$ -intercepts to prove our thinking? How about a graphing calculator?
  - *Begin your discussion by focusing on scholars who expanded the equations and identified the  $y$ -intercepts as a key way to determine which graph matches which equation. Once scholars have convinced each other that the first two equations match the graphs, discuss how the third equation is similar and different to the first and ask scholars to predict in what ways the graph will be similar and different to the graph of the first equation.*
  - *Then give scholars time to graph this equation in their graphing calculators to test their predictions before asking them to compare and interpret the  $x$ -intercepts of these graphs.*
- How do the  $x$ -intercepts of these graphs compare?
- What do the  $x$ -intercepts of each graph represent?
- What would need to be true about the factors in order for the equation to equal 0? Why?
- How are the factors and factored form of these equations related to the  $x$ -intercepts of these graphs?
- When graphing a quadratic function would you prefer for the function to be written in standard form or factored form? Why?

### Example 2



The purpose of this Mini-lesson is to consider what the graph of a quadratic function that is a perfect square when factored looks like and why. Scholars should leave the Mini-lesson understanding that this is a unique type of quadratic function that only touches the  $x$ -axis, but doesn't cross it. Because both factors are the same, there is only one value that makes the output zero. Therefore this function has just one  $x$ -intercept, which is also the vertex.

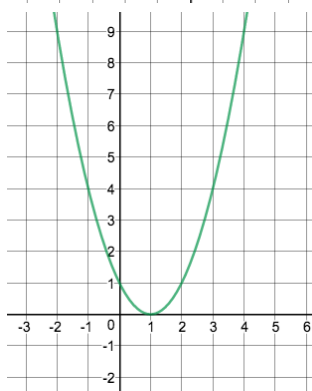
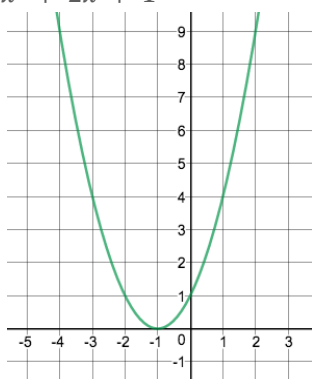
Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class. To review what scholars learned in Lesson 3, you may begin by showing just the factored form of the first equation  $f(x) = (x - 1)(x - 1)$  and asking scholars what predictions they can make about the graph based on that equation before revealing the other representations.

### Representations

$$f(x) = (x - 1)^2$$

$$f(x) = x^2 + 6x + 9$$

$$f(x) = x^2 + 2x + 1$$



### Key Questions

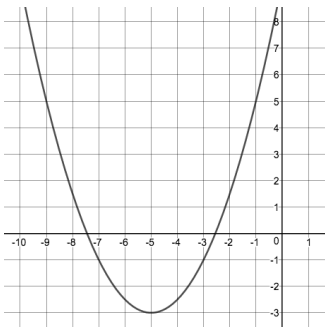
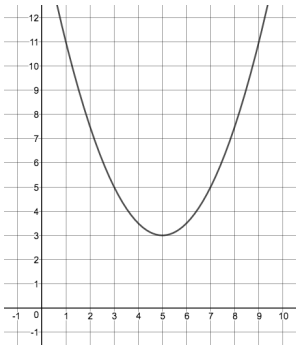
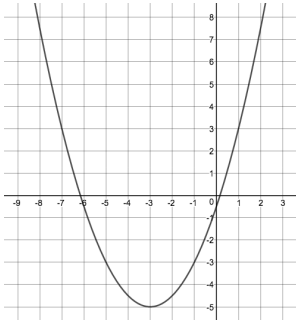
- How do you know these two representations match?
- How can we use our understanding of factoring and  $x$ -intercepts to prove our thinking? How about a graphing calculator?
  - *As scholars discuss the fact that there is only one  $x$ -intercept, be sure to define the term **vertex** as the minimum or maximum point of a parabola. Ask scholars how they can use either the standard or factored form of the function to decide whether the vertex will be a minimum or maximum.*
- How are these functions similar to and different than the functions we explored yesterday?
- What would the parabola of the final function look like? How do you know?

### Example 3

The purpose of this Mini-lesson to reinforce scholars' understanding of the relationship between a graph of a quadratic function and the algebraic representation of the same function in vertex form.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

### Representations



$$f(x) = \frac{1}{2}(x + 5)^2 - 3$$

$$f(x) = \frac{1}{2}(x + 3)^2 - 5$$

### Key Questions

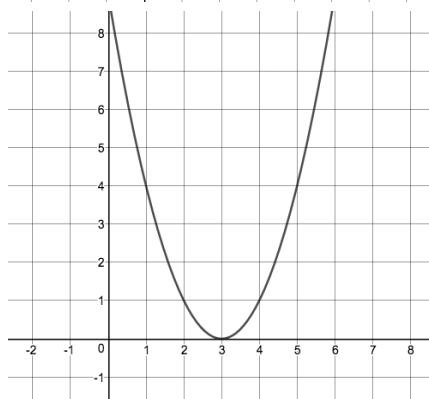
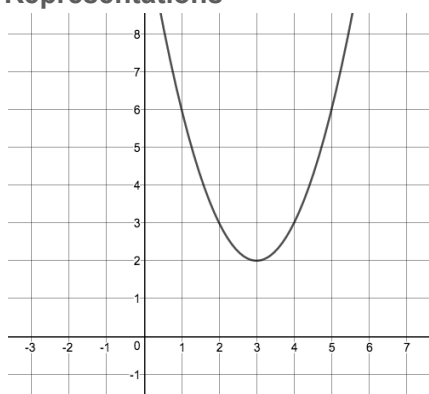
- How do you know these two representations match?
- Without trying to memorize which coordinate of the vertex is  $h$  and which is  $k$  in vertex form, how can we prove our thinking? How about with a graphing calculator?
- How can you prove that the leading coefficient (the  $\frac{1}{2}$ ) in this equation is accurate?
- What might the equation for the final function be in vertex form?
- Do we have enough information to write this function exactly?
- What did you learn about quadratic functions from this Mini-lesson?

#### Example 4

Scholars should walk away from this Mini-lesson recognizing that not all quadratic functions are factorable. Graphically, scholars should recognize that a quadratic function is not factorable if the function does not cross the  $x$ -axis and therefore does not have any (real) zeros. Algebraically, scholars should understand that if there are no terms that add together to get the  $b$  term of the standard equation and simultaneously multiply together to get the  $c$  term of the standard equation the function is not factorable. Through this Mini-lesson scholars should recognize that one of the benefits of vertex form is that, like standard form, it can be used to algebraically represent **any** quadratic function.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

#### Representations



$$y = (x - 3)^2 - 1$$

$$y = (x - 3)^2$$

$$y = (x - 3)^2 + 2$$

#### Key Questions

- How do you know these two representations match?
- How can we use our understanding of vertex form to prove our thinking? How about a graphing calculator?
- What would the graph of the final equation look like? How do you know?
- Could all of these functions be written in factored form? Why or why not?
- Could all of these functions be written in standard form? Why or why not?

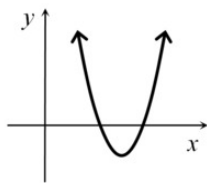
- What would the parabola of the final function look like? How do you know?
- What did you learn about quadratic functions from this Mini-lesson?

### Example 5

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

This version of Connecting Representations is different than what scholars are used to. Instead of having approximately the same number of graphs and equations, this time they are given one graph with no scale and asked to determine which equations **could** match. The purpose of this Mini-lesson is for scholars to practice using different forms of quadratic functions to identify key features of a graph as well as to build reasoning skills.

### Representations



$f_1(x) = (x + 12)^2 + 4$	$f_5(x) = -4(x + 2)(x + 3)$
$f_2(x) = -(x - 2)^2 - 1$	$f_6(x) = (x + 4)(x - 6)$
$f_3(x) = (x + 18)^2 - 40$	$f_7(x) = (x - 12)(-x + 18)$
$f_4(x) = (x - 12)^2 - 9$	$f_8(x) = (24 - x)(40 - x)$

### Key Questions

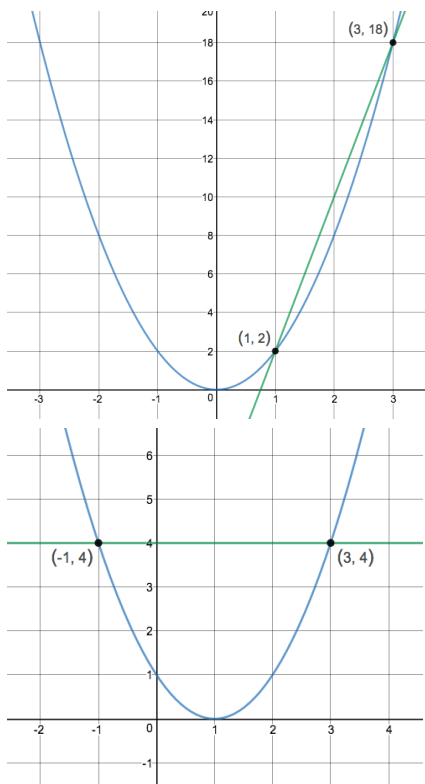
- Which equations *could* match this graph? How do you know?
- Are we going to be able to prove that these equations match this graph exactly? Why or why not?
- Are there any equations that you can immediately rule out as *not* matching the graph? Why?
- What information can you gain from looking at a quadratic function written in factored form compared to a quadratic function in vertex form?
- Is one form more helpful than the other for this task? Why or why not?

### Example 6

The purpose of this Mini-lesson is to strengthen scholars' understanding that any equation can be solved by graphing each side and finding where lines intersect. Scholars should use what they learned in the previous lesson to choose an efficient algebraic strategy to prove their thinking.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

### Representations



$$(x - 1)^2 = 4$$

$$3 = x^2 + 2x$$

$$2x^2 = 8x - 6$$

### Key Questions

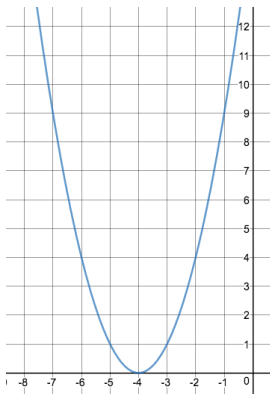
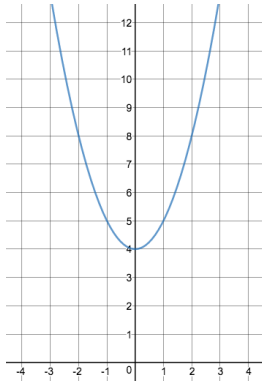
- How do you know these two representations match?
- How can we prove our thinking algebraically?
- What did you learn about solving quadratic equations from this Mini-lesson?
- How could we graphically find the solution to the final function?

### Example 7

The purpose of this Mini-lesson is to introduce scholars to the word **transformation** to describe how a function has changed from its original form. Scholars should be able to use graphs and tables to identify the how the function  $y = x^2$  was transformed. Scholars will recognize that going from  $x^2$  to  $x^2 + 4$  shifts the function up and going from  $x^2$  to  $(x + 4)^2$  shifts the function to the left.

Put up all of the representations. The goal is for students to determine which representations are connected (i.e., which equations match which graphs). Scholars should have an opportunity to consider the representations individually and then in partners before discussing as a whole class.

### Representations



$$y = x^2$$

$$y = x^2 + 4$$

$$y = (x + 4)^2$$

### Key Questions

- How do you know these two representations match?
- How can we use a table to prove our thinking? How about a graphing calculator?
- How are these representations similar? How are they different?
- If we think of  $y = x^2$  as our original function, how would you describe how the function has been changed or transformed with each new equation?