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Middle School Math: Guide to Number Strings

Number Strings are Mini-lessons that enhance scholars' numeracy, fluency, and confidence with math. When executed at a high level, Number Strings encourage scholars to visualize numbers and operations and to choose strategies that make problems friendlier. Number Strings look deceptively simple, but they require careful planning and flexible execution. This document highlights best practices for planning and implementing Number Strings.

Planning for Number Strings:

Step 1: Choose the one goal you will drive towards in this Mini-lesson.

- Solve all problems to understand the relationships between them.
- Identify the relationships in this string that will help scholars solve other problems in the future.
- Use trends in scholar work and misconceptions to determine **one high leverage takeaway** that you will use this string to drive towards. What do you want scholars to say when you ask them for this takeaway at the end of the string?
- What will you look for in scholar work to know if they are applying this takeaway to future problems?

Step 2: Determine how you will represent scholar thinking visually to drive toward your goal.

- What model or models will you use to represent scholars' thinking in order to highlight key relationships?
- Practice drawing models for each problem so that you have a sample that shows what you want your chart paper to look like when you are done.

Step 3: Plan your launch and context.

- What story or context will naturally encourage scholars to imagine the model you want scholars to use?
- How will you tell this story succinctly? How will you set scholars up to use the story and model to make sense of the numbers in the string?
- **Notes on contexts for strings:**
  - Contexts for strings must describe situations that scholars can **imagine** and **envision**. They should be fairly simple.
  - Contexts support scholars in exploring and developing new concepts, models, and strategies. Contexts help convince scholars that the relationships they see are valid.
    - For example, a scholar can envision a box with 8 rows of 14 chocolates each as two boxes with 4 rows of 14 chocolates each put together. This supports scholars with to see why 8 × 14 is equivalent to (4 × 14) + (4 × 14).

Step 4: Plan your pacing.
- Identify the questions in the string where scholars will meet your goal. Plan to spend the most time on these questions.
- Plan to move quickly through "helper questions" that you expect will be easy for scholars.
- Identify repetitive groups of problems so that you can skip these if you are pressed for time.

Best Practices for Implementing Number Strings:
Implementing successful number strings requires flexibility. Well-planned strings fall flat when teachers inflexibly rely on pre-scripted plans without responding in the moment to what scholars actually say. Evaluate whether you are making progress toward your goal by listening authentically to scholars and asking questions that focus their thinking about the concepts, models, or strategies you are working to develop.

Ask; do not tell.
Asking questions requires scholars to do most of the talking, and thus most of the thinking.
- Ask scholars how they are visualizing the problems, and represent their thinking for the class as they share. Be strategic and ask follow up questions to clarify how and why the visual models represent the problems.
- Ask scholars to confirm or reject answers based on each other's justifications. Do not confirm answers as the teacher.
- Ask if anyone else solved the problem using a different strategy. Have scholars re-state each other's strategies in their own words.

Focus on the thinking.
Number Strings include problems that all scholars could solve with pencil and paper, so getting to the right answer is a very small part of this routine. Agreeing on an answer must be the beginning of the conversation, not the end.
- Ask how scholars came to their answers and how to show their strategies visually.
- Value multiple approaches. When comparing multiple strategies, listen during turn and talks and strategically call on scholars who's thinking will help you drive toward your goal.
- Celebrate skeptical scholars! Press others to convince skeptics by explaining their thinking in new ways.
- Play the skeptic yourself. If you do not hear or see misconceptions that you anticipated, introduce these to the conversation as your own strategies and ask scholars to evaluate them.

Explicitly question scholars about relationships between problems.
In isolation, the problems in most strings are neither challenging nor interesting. The point is for scholars to compare different problems and strategically manipulate challenging problems to make them friendlier.
- Ask scholars how to use a previous problem or model to solve the next one.
- Ask scholars what pattern they notice in this string.
- Ask scholars to predict what problem might come next.
- Say, "You're never going to see this exact string again. What are you taking away to help you with future math problems?"

Move quickly to keep engagement up.

Not all questions will take the same amount of time. Plan for which questions are worth spending time on. Strategically ask for multiple answers or strategies on challenging or high-leverage problems.

Sample Number Strings

Adapted from Context For Learning: Minilessons for Early Multiplication and Division and Minilessons for Extending Multiplication and Division

Number Strings

In a Number String, your job is to put up the problems one at a time, then ask scholars to share how they solved, and represent their thinking on a visual model for the whole class to see. As scholars share their thinking, press them to tell you exactly what they are envisioning on the visual model so that you can represent their thinking accurately. Keep the problems and visual models displayed as you progress through the string so that scholars can make connections to previous problems and models.

Multiplication Number Strings

As you work through this string, represent scholar strategies with arrays and equations. When multiplying a single-digit number and a multi-digit number, it's often helpful to multiply the single digit by the value of each digit in the larger number, and then add the products. For instance,

129×8 can be solved by 100×8, 20×8, 9×8, and then adding the products. The goal is for scholars to understand why this works and why it's helpful, which will set them up to understand the standard multiplication algorithm.

$$100×8$$

$$109×8$$

$$129×8$$

$$200×8$$

$$4×8$$

$$60×8$$

264×8

304×5

As you work through this string, represent scholar strategies with arrays and equations. Emphasize the idea that a certain strategy can be represented using different models. (For instance, the same partial products strategy for 11×13 could be shown with an equation as $(10×10)+(10×3)+(1×10)+(1×3)=143$, or as an array that models breaking up and multiplying the same way.) Understanding this idea will set scholars up for success in the discourse, when they must recognize similarities between the standard algorithm and other methods of multiplication.

10×13

11×13

9×13

20×13

21×13

19×13

101×13

Division Number Strings

Work through the string below, representing scholar strategies on the array. Focus on partial quotients strategies, and why they work. This will set scholars up to use partial quotients on larger numbers during the Explore, and to think about this strategy deeply in preparation for the Discourse.

21÷3

30÷3

51÷3

81÷3

140÷14

154÷14

280÷14

4

294÷14

Division Number Strings

Work through the string below, representing scholar strategies on the array. Allow scholars to debate the best strategy for various problems: Are partial quotients or multiplication easier? What's the best way to break the numbers down? How helpful is an array? Scholars should have mastered these strategies and models. Now they must think strategically about when to deploy each one.

28÷7

70÷7

98÷7

168÷7

170÷17

187÷17

340÷17

357÷17

Multiplication and Division Number Strings

The goal of this string is to use the relationship between multiplication and division to illuminate different interpretations of division.

In each problem in this string, Kara the kangaroo is jumping along a track. Scholars must determine the number of jumps needed to reach the target given the length of the kangaroo's jump and the total distance to the target.

___×12=60

 – Kara jumps by 12's and the target is located at 60 on the track. How many jumps does she need to take to reach the target?

___×6=60

– What if Kara jumps by 6's? *"How can we use our last picture to help us answer this?"*

6×____=60

– What if I know Kara took 6 jumps to get to 60 instead? *"How does this change the picture?"*

6÷____=6 – *"What could this question mean in the context of our story?"*

____×5=60

– If Kara jumps by 5's, how many jumps does it take to get to 60? *"Which picture can you use to help you answer this question?"*

60÷____=5 – *"What could this question represent in our story? How does the picture change?"*
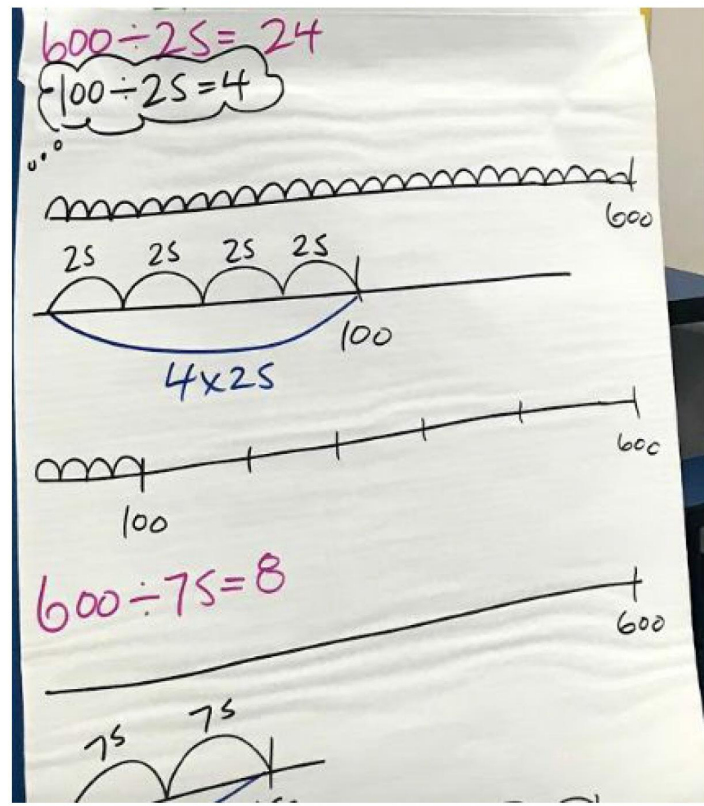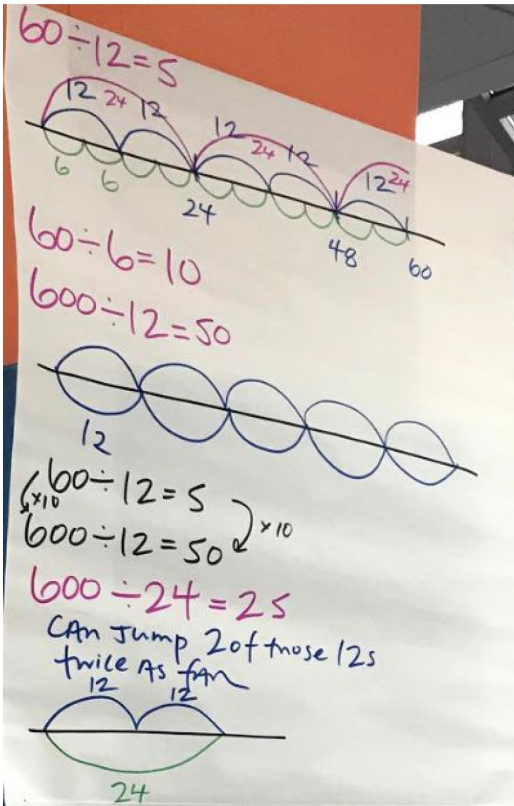
____×12=600 – If Kara jumps by 12's, how many jumps does it take to get to 600?

12×____=600 – What if Kara takes `1 jumps? *"Which situation is easier for you to picture? Why?"*

6

600÷____=24 – *"What might this question mean in our story?"*

600÷____=25 – *"How can we use the previous question to help us answer this one?"*

600÷____=75

Number String

The goal of this string is for scholars to understand that scaling the dividend and divisor by the same amount does not change the size of the quotient.

24÷4 – "How many groups of 4 are in 24? How can we draw this?"

6÷1 – "How many groups of 1 are in 6? How is our model for this question similar to or different than the last one?"

2÷13 – "How many groups of 13 are in 2? How is this model similar to or different than the last one?"

4÷13 – "Will the quotient be larger or smaller than the last question? How do you know?

3÷14 – "How is this model similar to or different than the last one?"

3÷24 – "Will this quotient be larger or smaller than the last one?

3÷34 – "Will this quotient be greater than 1or less than 1?" and "Which previous problem helps you the most with this one?"

6÷14 – "Which diagram can you use to help you answer this question?"

6÷13 – "Which diagram can you use to help you answer *this* question?"

6÷23

4÷25 – "What friendlier problem would you solve first to help you answer this question?"

$24 \div 4$

$6 \div 1$

24

6

$2 \div 1/3$ How many $1/3$s are in 2?

| 1/3 | 1/3 | 1/3 | 1/3 | 1/3 | 1/3 |

$4 \div 1/3$

1/3 2/3 1   4/3 5/3 2
            3/3   6/3

2

$3 \div \frac{1}{4} = 12$

2/4   2/4   1

$3 \div 2/4 = 6$

$3 \div 3/4 = 4$

3/4   1   2   3

$6 \div 1/4$

$6 \div 1/3$

$6 \div 2/3$

1/3 1/2 1/2

1